Cheat Sheet

UNIT-3

1. $\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt = F(s),\ s = \sigma + i\omega > 0$

2. $\mathcal{L}\{f(at)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$

3. $\mathcal{L}\left\{f\left(\frac{t}{a}\right)\right\} = a\,F(as)$

4. $\mathcal{L}\{k\} = \frac{k}{s}$

5. $\mathcal{L}\{e^{at}\} = \frac{1}{s-a},\ s > a$

6. $\mathcal{L}\{e^{-at}\} = \frac{1}{s+a}$

7. $\mathcal{L}\{\sinh at\} = \frac{a}{s^2-a^2}$

8. $\mathcal{L}\{\cosh at\} = \frac{s}{s^2-a^2}$

9. $\mathcal{L}\{a^t\} = \frac{1}{s-\ln a}$

10. $\mathcal{L}\{\sin at\} = \frac{a}{s^2+a^2}$

11. $\mathcal{L}\{\cos at\} = \frac{s}{s^2+a^2}$

12. $\mathcal{L}\{t^n\} = \frac{\Gamma(n+1)}{s^{n+1}}$

13. $\cosh(a \pm b) = \cosh a \cosh b \pm \sinh a \sinh b$

14. $\mathcal{L}\{\sin\sqrt{t}\} = \frac{1}{2s}\sqrt{\frac{\pi}{s}}\, e^{-1/4s}$; $\mathcal{L}\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\} = \sqrt{\frac{\pi}{s}}\, e^{-1/4s}$

15. $\mathcal{L}\{e^{at} f(t)\} = F(s-a)$

16. $\mathcal{L}\left\{\frac{\sin t}{t}\right\} = \cot^{-1} s$

17. $\int e^{ax}\cos bx = \frac{e^{ax}}{a^2+b^2}[a\cos bx + b\sin bx]$

18. $\int e^{ax}\sin bx = \frac{e^{ax}}{a^2+b^2}[a\sin bx - b\cos bx]$

19. $\mathcal{L}\{u(t)\} = \frac{1}{s}$

homework: learn all formulas

20. $\mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s}$

21. $\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}F(s)$

22. $\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n(F(s))}{ds^n}$

23. $\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(s)\,ds$

24. $\mathcal{L}\{f'(t)\} = sF(s) - f(0)$

25. $\mathcal{L}\{f''(t)\} = -f'(0) - sf(0) + s^2F(s)$

26. $\mathcal{L}\left\{\int_0^t f(t)\,dt\right\} = \frac{F(s)}{s}$

27. $\mathcal{L}\left\{\underbrace{\int\int\cdots}_{n} f(t)\,\underbrace{dt\,dt\,dt}_{n}\right\} = \frac{F(s)}{s^n}$

28. $\mathcal{L}\{f(t)\}$ (periodic) $= \frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)\,dt$

29. $\mathcal{L}\{\delta(t-a)\} = e^{-as}$

30. $\mathcal{L}\{\delta(t-a)\} = \mathcal{L}\{u'(t-a)\}$

31. $\mathcal{L}\{\delta(t)\} = 1$

32. $\mathcal{L}\{f(t)\,\delta(t-a)\} = e^{-as} f(a)$

33. $\int e^{ax} \cos bx \, dx = \dfrac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$

34. $\int e^{ax} \sin bx \, dx = \dfrac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$

UNIT-4

1. $\mathcal{L}^{-1}\left\{\frac{k}{s}\right\} = k$

2. $\mathcal{L}^{-1}\left\{\frac{1}{s^n}\right\} = \frac{1}{\Gamma(n)} t^{n-1}$ or $\frac{t^{n-1}}{(n-1)!}$

3. $\mathcal{L}^{-1}\left\{\frac{1}{s-a}\right\} = e^{at}$

4. $\mathcal{L}^{-1}\left\{\frac{1}{s+a}\right\} = e^{-at}$

5. $\mathcal{L}^{-1}\left\{\frac{1}{s-c\ln b}\right\} = e^{c\ln b\, t} = b^{ct}$

6. $\mathcal{L}^{-1}\left\{\frac{1}{s^2+a^2}\right\} = \frac{1}{a}\sin(at)$

7. $\mathcal{L}^{-1}\left\{\frac{s}{s^2+a^2}\right\} = \cos(at)$

8. $\mathcal{L}^{-1}\left\{\frac{1}{s^2-a^2}\right\} = \frac{1}{a}\sinh(at)$

9. $\mathcal{L}^{-1}\left\{\frac{s}{s^2-a^2}\right\} = \cosh(at)$

10. $\mathcal{L}^{-1}\left\{\frac{1}{(s\mp a)^2+b^2}\right\} = \frac{e^{\pm at}}{b}\sin(bt)$

11. $\mathcal{L}^{-1}\left\{\frac{(s\mp a)}{(s\mp a)^2+b^2}\right\} = e^{\pm at}\cos(bt)$

12. $\mathcal{L}^{-1}\left\{F^n(s)\right\} = (-1)^n t^n f(t)$ ← n^{th} derivative

13. $\mathcal{L}^{-1}\left\{\int_s^\infty \int_s^\infty \cdots \int_s^\infty F(s)\,ds \cdots ds\,ds\right\} = \frac{f(t)}{t^n}$

14. $\mathcal{L}^{-1}\left\{\frac{F(s)}{s^n}\right\} = \int_0^\infty \int_0^\infty \cdots f(t) \cdots dt\,dt$

15. $\mathcal{L}^{-1}\left\{sF(s)\right\} = f'(t)$ iff $f(0)=0$

16. $\mathcal{L}^{-1}\left\{1\right\} = \delta(t)$

17. $\mathcal{L}^{-1}\left\{e^{-as}\right\} = \delta(t-a)$

18. $\mathcal{L}^{-1}\left\{\frac{e^{-as}}{s}\right\} = u(t-a)$

19. $\mathcal{L}^{-1}\left\{e^{-as}F(s)\right\} = f(t-a)\,u(t-a)$

20. $\mathcal{L}^{-1}\{e^{-as}f(s)\} = f(t)\,\delta(t-a)$

* 21. $\mathcal{L}^{-1}\left\{\frac{1}{\sqrt{s}}\right\} = \frac{1}{\sqrt{\pi t}}$

* 21. $\mathcal{L}^{-1}\left\{\frac{1}{s\sqrt{s}}\right\} = 2\sqrt{\frac{t}{\pi}}$

* 22. $\mathcal{L}^{-1}\left\{\frac{1}{(s^2+a^2)^2}\right\} = \int_0^t \frac{t}{2a}\sin at\,dt = \frac{1}{2a^2}\left(\frac{\sin at - t\cos at}{a}\right)$

* 23. $\mathcal{L}^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\} = \frac{1}{2a}t\sin at$

24. $\mathcal{L}^{-1}\left\{\frac{1}{(s\mp a)^n}\right\} = \frac{e^{\pm at}t^{n-1}}{\Gamma(n)}$

* 25. $\mathcal{L}^{-1}\left\{\frac{s^2-a^2}{(s^2+a^2)^2}\right\} = e^{-at}$

26. $\mathcal{L}^{-1}\left\{\frac{1}{s\mp a - c\ln b}\right\} = e^{\pm at}b^{ct}$

27. $\mathcal{L}^{-1}\left\{\frac{1}{(s\mp a)^2-b^2}\right\} = \frac{e^{\pm at}}{b}\sinh bt$

28. $\mathcal{L}^{-1}\left\{\frac{s\mp a}{(s\mp a)^2-b^2}\right\} = e^{\pm at}\cosh bt$

29. $\mathcal{L}^{-1}\{F(s \mp a)\} = e^{\pm at} f(t)$